1998 Semester 1 Multiple Choice

1. What is the x-coordinate of the point of inflection on the graph of $y=\frac{1}{3}x^3+5x^2+24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

6. If $x^2+xy=10$, then when $x=2$, $\frac{dy}{dx}=$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

8. Let f and g be differentiable functions with the following properties:

(i) $g(x)>0$ for all x
(ii) $f(0)=1$

If $h(x)=f(x)g(x)$ and $h'(x)=f(x)g'(x)$, then $f(x)=$

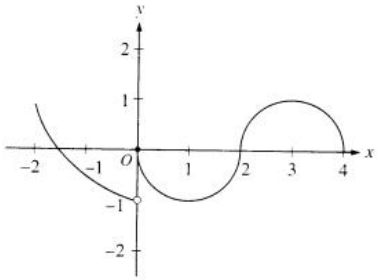
(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

10. What is the instantaneous rate of change at $x=2$ of the function f given by $f(x)=\frac{x^2-2}{x-1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

12. If $f(x)=\begin{cases}\ln x & \text{for } 0<x\le 2\\ x^2\ln 2 & \text{for } 2<x\le 4,\end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2<x<4$, is f not differentiable?

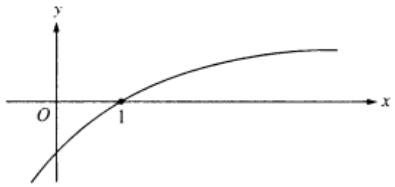
(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14. A particle moves along the x-axis so that its position at time t is given by $x(t)=t^2-6t+5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

16. If $f(x)=\sin\left(e^{-x}\right)$, then $f'(x)=$

(A) $-\cos(e^{-x})$
(B) $\cos(e^{-x})+e^{-x}$
(C) $\cos(e^{-x})-e^{-x}$
(D) $e^{-x}\cos(e^{-x})$
(E) $-e^{-x}\cos(e^{-x})$



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1)<f'(1)<f''(1)$
(B) $f(1)<f''(1)<f'(1)$
(C) $f'(1)<f(1)<f''(1)$
(D) $f''(1)<f(1)<f'(1)$
(E) $f''(1)<f'(1)<f(1)$

18. An equation of the line tangent to the graph of $y=x+\cos x$ at the point $(0,1)$ is

(A) $y=2x+1$ (B) $y=x+1$ (C) $y=x$ (D) $y=x-1$ (E) $y=0$

19. If $f''(x)=x(x+1)(x-2)^2$, then the graph of f has inflection points when $x=$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

22. The function f is given by $f(x)=x^4+x^2-2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}},\infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

(C) $(0,\infty)$

(D) $(-\infty,0)$

(E) $\left(-\infty,-\frac{1}{\sqrt{2}}\right)$

24. The maximum acceleration attained on the interval $0\le t\le 3$ by the particle whose velocity is given by $v(t)=t^3-3t^2+12t+4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

x	0	1	2
$f(x)$	1	k	2

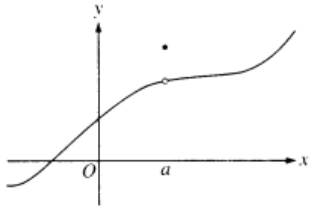
26. The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x)=\frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k=$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

28. If $f(x)=\tan(2x)$, then $f'\left(\frac{\pi}{6}\right)=$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

Graphing Calculator Allowed



76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x=a$.

(B) f has a relative maximum at $x=a$.

(C) $x=a$ is in the domain of f.

(D) $\lim_{x\to a^+} f(x)$ is equal to $\lim_{x\to a^-} f(x)$.

(E) $\lim_{x\to a} f(x)$ exists.

77. Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) –0.701
(B) –0.567
(C) –0.391
(D) –0.302
(E) –0.258

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

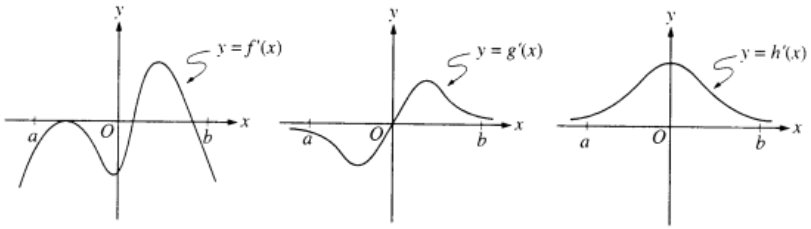
(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2\pi C$



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

80. The first derivative of the function f is given by $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

83. If $a \neq 0$, then $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

91. Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III